International Journal of Materials Science and Applications
2023; 12(1): 1-7
http://www.sciencepublishinggroup.com/j/ijmsa
doi: 10.11648/j.ijmsa.20231201.11
ISSN: 2327-2635 (Print); ISSN: 2327-2643 (Online)

A Rheological Model with Integer and Non-Integer Orders Nonlinearities for Predicting Stress Relaxation Behavior in Viscoelastic Materials

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To cite this article:

Yélomè Judicaël Fernando Kpomahou, Koffi Judicaël Agbélélé, Arnaud Edouard Yamadjako, Bachir Koladé Adélakoun Ambelohoun. A Rheological Model with Integer and Non-Integer Orders Nonlinearities for Predicting Stress Relaxation Behavior in Viscoelastic Materials. *International Journal of Materials Science and Applications*. Vol. 12, No. 1, 2023, pp. 1-7. doi: 10.11648/j.ijmsa.20231201.11

Received: November 21, 2022; **Accepted**: December 13, 2022; **Published**: March 4, 2023

Abstract: Viscoelastic materials are widely used as devices for vibration control in modern engineering applications. They exhibit both viscous and elastic characteristic when undergoing deformation. They are mainly characterized by three time-dependent mechanical properties such as creep, stress relaxation and hysteresis. Among them, stress relaxation is one of the most important features in the characterization of viscoelastic materials. This phenomenon is defined as a time-dependent decrease in stress under a constant strain. Due to the inherent nonlinearity shown by the material response over a certain range of strain when viscoelastic materials are subjected to external loads, nonlinear rheological models are needed to better describe the experimental data. In this study, a singlenonlinear differential constitutive equation is derived froma nonlinear rheological model composed of a generalized nonlinear Maxwell fluid model in parallel with a nonlinear spring obeying a power law for the prediction of the stress relaxation behavior in viscoelastic materials. Under a constant strain-history, the time-dependent stress is analytically derived in the cases where the positive power law exponent, $\alpha < 1$ and $\alpha > 1$. The Trust Region Method available in MATLAB Optimization Toolbox is used to identify the material parameters. Significant correlations are found between the experimental relaxation data taken from literature and exact analytical predictions. The obtained results show that the developed rheological model with integer and non-integer orders nonlinearities accurately describes the experimental relaxation data of some viscoelastic materials.

Keywords: Nonlinear Rheological Model, Integer and Non-Integer Orders Nonlinearities, Stress Relaxation, Viscoelastic Materials

1. Introduction

Real materials, subjected to different external loads, can behave like liquids or solids. This behavior, which ranges between liquid and solid, is referred to as the viscoelastic behavior or material response. In consideration of the material response, the real materials which are viscoelastic materials are nowadays widely used because of their interesting viscoelastic properties. The time-dependent behavior appears when the viscoelastic materials are subjected to various loading conditions. For instance, under a constant stress and constant strain, the viscoelastic materials exhibit a slow continuous increase in strain over time, so-called creep and a slow continuous decrease in stress with time, so-called stress relaxation respectively. These phenomena are inherent to viscoelastic materials. They play an important role in many scientific fields, such as engineering medical, foods engineering, biological and biomechanical research, etc. Therefore, to better understand and quantify viscoelastic properties, rheological models consisting of springs and dashpots are needed, since these models with few elements describe the behavior of viscoelastic materials in simpler terms [1, 2]. To this end, classical rheological models, such as Maxwell, Kelvin-Voigt, Zener, Burger models, etc., are used to quantify the viscoelastic properties [3, 4]. Moreover, more complex linear combinations are widely used to describe the viscoelastic behavior of the real materials [5-8].

Actually, the time-dependent properties of the real materials cannot be completelyrepresented by linear rheological models [9, 10]. The main reason is that, these materials exhibit a large deformation even under a small strain or stress history. Therefore, the well-known linear viscoelastic theory may be substituted by nonlinear viscoelastic theory. It is interesting to note that the nonlinear viscoelastic theory has no definitive formulation. For this reason, several nonlinear viscoelastic theories of different complexities have been developed in an integral or differential form. Nonlinear integral formulations are widely used in the open literature by many investigators for describing the time-dependent properties of the biological materials. However these nonlinear integral theories provide phenomenological models with time-dependent parameters which are without physical meaning [6, 11]. On the one other hand, the nonlinear integral forms of constitutive relations are often difficult to solve and to simulate and, on the other hand, they present a limited range of applicability governed by experimental data which is difficult to define [2]. Most nonlinear viscoelastic models for describing satisfactorily the time-dependent properties of the real materials are often based on the differential formulations. These latter are based on the extension of the classical rheological models to the finite deformations. Following this technique, a simplified nonlinear model consisted of a Voigt element in series with a nonlinear dashpot obeying a power law has been used for describing the viscoelastic properties of wheat flour doughs at high shear strain [12]. In [10], the authors considered a classical Voigt model in parallel with a system of nonlinear springs to model the viscoelastic materials behavior. In this perspective, a nonlinear generalized Maxwell fluid model consisting of a linear dashpot in series with a parallel arrangement of a linear spring and a second-order nonlinear spring has been developed for describing both the stiffening and softening response of some viscoelastic materials [11]. Recently in [13], a second-order elastic spring in series with a classical Voigt element has been developed for reproducing the nonlinear time-dependent stress response of some viscoelastic materials. More recently, a standard nonlinear solid model consisting of a polynomial elastic spring in series with a classical Voigt model and modified Voigt model have been proposed for predicting the nonlinear time-dependent stress induced in some viscoelastic materials [14, 15]. More recently again, a nonlinear spring in series with a nonlinear dashpot obeying a power-law has been used for representing successfully the nonlinear time-dependent stress response of some viscoelastic materials [16].

It appears significant to signal that, there are very few nonlinear rheological models developed with constant-value material parameters capable to predict the nonlinear stress relaxation behavior in some viscoelastic materials. Thus, the problem that arises is then to develop nonlinear rheological models with constant-value material coefficient for predicting the nonlinear stress relaxation behavior in some viscoelastic materials. So, the research question that deserves to be asked here is the following: Does a rheological model with integer and noninteger orders nonlinearities is able to perfectly reproduce the stress relaxation data of viscoelastic materials? In order to answer this question, we presume that the stress relaxation data of the viscoelastic materials can be described by a nonlinear rheological model.

To verify this assumption, we firstly develop a nonlinear rheological model consisted of a nonlinear Maxwell fluid model in parallel with a nonlinear spring element for predicting the nonlinear stress relaxation behavior in viscoelastic materials (Section 2). Secondly, we perform numerical simulations for validating the proposed nonlinear model (Section 3). Finally, we end with a conclusion (Section 4).

2. Model Formulation

2.1. Theoretical Consideration

To develop our proposed nonlinear rheological model shown in Figure 1, we have chosen the standard solid model [8] in which we replace the linear spring in parallel by a nonlinear spring with stiffness E_β obeying a power law. Afterward, we add in parallel to the linear spring with stiffness E_M which is in series with linear dashpot with viscous damping η_M, a nonlinear spring with stiffness E_α obeying equally a power law. Thus, in this combination, the nonlinear springs with stiffness E_α and E_β allow capturing the nonlinear viscoelastic properties of the materials while the Maxwell fluid element capture the linear behavior of the materials.

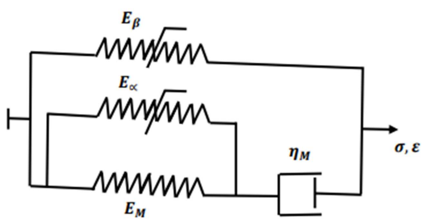


Figure 1. *Proposed nonlinear rheological model.*

From proposed nonlinear rheological model, the total stress $\sigma\ (t)$ of the materials under consideration is given by

$$\sigma = \sigma_1 + \sigma_2 \tag{1}$$

where σ_1 and σ'' represent the stress of the nonlinear spring with stiffness E_β and the nonlinear generalized Maxwell fluid model respectively. In addition, the total strain $\varepsilon\ (t)$ of the material is given by the following relationship:

$$\varepsilon = \varepsilon' = \varepsilon'' \tag{2}$$

where ε_1 is the strain of the nonlinear springs with stiffness E_β and ε'' designates the strain of the nonlinear generalized Maxwell fluid model. Due to the fact that the linear dashpot is in series with an association of two springs in parallel, the strain ε_2 can be written as follows:

$$\varepsilon_2 = \varepsilon = \varepsilon_3 + \varepsilon_{\eta M} \tag{3}$$

where ε_3 represents the strain of the equivalent spring composed ofa linear and nonlinear spring and $\varepsilon_{\eta M}$ designates the strain of the linear dashpot. The stress σ_1 in nonlinear spring with stiffness E_β and σ_2 in the nonlinear generalized Maxwell fluid model are respectively defined as

$$\sigma_1 = E_\beta \varepsilon^\beta \tag{4}$$

and

$$\sigma_2 = E_M \varepsilon_3 + E_\alpha \varepsilon_3^\alpha = \eta_M \dot{\varepsilon}_{\eta M} \tag{5}$$

where α and β denotes the power-law exponent and the dot means the time derivative. Now, using Eqs. (4) and (5), the expression of the total stress in the considered materials can be written as follows

$$\sigma = E_M \varepsilon_3 + E_\alpha \varepsilon_3^\alpha + E_\beta \varepsilon^\beta \tag{6}$$

or

$$\sigma = E_\beta \varepsilon^\beta + \eta_M \dot{\varepsilon}_{\eta M} \tag{7}$$

From Eqs. (3) and (5), we derive the following equation

$$\dot{\varepsilon}_3 = -\frac{1}{\tau_\alpha}\varepsilon_3^\alpha - \frac{1}{\tau_M}\varepsilon_3 + \dot{\varepsilon} \tag{8}$$

where $\tau_\alpha = \dfrac{\eta_M}{E_\alpha}$ and $\tau_M = \dfrac{\eta_M}{E_M}$ represent the relaxation parameters. Thus, Eq. (8) also obtained represents mathematically, in the single differential form, the relation between the total strain $\varepsilon(t)$ and the strain $\varepsilon_3(t)$ of the equivalent spring. This equation is the first-order nonlinear ordinary differential equation in ε_3 for a given strain history $\varepsilon(t)$.

At present, we study the nonlinear stress relaxation behavior in viscoelastic materials in the following subsection.

2.2. Nonlinear Stress Relaxation Behavior

In the stress-relaxation test, it is well-known that the material sample is subjected at time to a suddenly applied strain which is maintained constant thereafter. Thus the applied strain $\varepsilon(t)$ has the following from

$$\varepsilon(t) = \begin{cases} 0 & if\ t \prec 0 \\ \varepsilon_0 & if\ t \geq 0 \end{cases} \tag{9}$$

$$\text{or}\ \varepsilon(t) = \varepsilon_0 H(t) \tag{10}$$

where ε_0 is a constant and $H(t)$ is the so called Heaviside unit function. Thus, following this strain-history, Eq. (8) becomes

$$\dot{\varepsilon}_3 = -\frac{1}{\tau_\alpha}\varepsilon_3^\alpha - \frac{1}{\tau_M}\varepsilon_3 \tag{11}$$

The equation (11) is also a first-order nonlinear ordinary differential equation which can be solved analytically using the initial condition:

$$t = 0, \varepsilon_3(t) = f_0$$

Thus, by performing some mathematical operations onto Eq. (11), we can obtain as solution

$$\varepsilon_3(t) = \left[\frac{\tau_M}{\tau_\alpha} + \left(f_0^{1-\alpha} - \frac{\tau_M}{\tau_\alpha}\right)\exp\left(-\frac{1-\alpha}{\tau_M}t\right)\right]^{\frac{1}{1-\alpha}} \tag{12}$$

Now, from Eq. (6), we can deduce by taking into account Eq. (12), two nonlinear stress relaxation responses in the materials under consideration. So, in the case where 0< α <1, the nonlinear stress relaxation behavior in the viscoelastic materials is described by the following nonlinear equation

$$\sigma(t) = E_M\left(g_0 + g_1\exp\left(-\frac{t}{\tau_0}\right)\right)\left(\frac{g_0}{2} + g_1\exp\left(-\frac{t}{\tau_0}\right)\right)^{\frac{\alpha}{1-\alpha}} + E_\beta\,\varepsilon_0^\beta \tag{13}$$

where the dimensionless parameters g_0 and g_1, and the relaxation time τ_0 are given by

$g_0 = 2\dfrac{E_\alpha}{E_M}$, $g_1 = f_0^{1-\alpha} - \dfrac{g_0}{2}$ and $\tau_0 = \dfrac{\tau_M}{1-\alpha}$ respectively.

The equation (13) describes the variation of the stress versus time as a function of the hyper-exponential type. The maximal stress in the material under consideration obtained at $t = 0$ is given by:

$$\sigma_0 = E_M f_0\left(1 + \frac{g_0}{2f_0^{1-\alpha}}\right) + E_\beta \varepsilon_0^\beta \tag{14}$$

From Eq. (14), we clearly see that at $t = 0$, the springs of elastic modulus E_M, E_α and E_β act on the material response. However, when the time tends to infinity, the residual stress, that is to say, the equilibrium stress verifies the following relationship.

$$\sigma_e = E_M g_0 \left(\frac{g_0}{2}\right)^{\frac{\alpha}{1-\alpha}} + E_\beta \varepsilon_0^\beta \tag{15}$$

We also note that the springs of elastic modulus E_M, E_α and E_β act when the time tends to infinity. From Eqs. (14) and (15), the difference between the maximal and residual stress given by

$$\sigma_i = E_M f_0 \left(1 + \frac{g_0}{2 f_0^{1-\alpha}}\right) - E_M g_0 \left(\frac{g_0}{2}\right)^{\frac{\alpha}{1-\alpha}} \tag{16}$$

is influenced by E_M, E_α, f_0 and α. At this level, the springs of elastic modulus E_M and E_α act on the material response.

When $\alpha>1$, the nonlinear stress relaxation response in the viscoelatic materials under study is governed by the following equation

$$\sigma(t) = E_M \frac{g_1 \exp\left(-\frac{t}{\tau_M}\right) + g_0 \exp\left(-\frac{t}{\tau_1}\right)}{\left(g_1 + \frac{g_0}{2}\exp\left(-\frac{t}{\tau_2}\right)\right)^{\frac{\alpha}{\alpha-1}}} + E_\beta \varepsilon_0^\beta \tag{17}$$

where τ_M, $\tau_1 = \frac{\tau_M}{\alpha}$ and $\tau_2 = \frac{\tau_M}{\alpha - 1}$ represent the three relaxation times. Note that from Eq. (17), the maximal stress in the material under consideration is given by Eq. (14). However, the equilibrium stress satisfies the following relationship:

$$\sigma_e = E_\beta \varepsilon_0^\beta \tag{18}$$

From Eq. (18), we observe that only the spring of elastic modulus E_β acts on the materials response when the time tends to infinity. Now, by taking into account Eqs. (14) and (18), the difference between the maximal and equilibrium stress is given by:

$$\sigma_i = E_M f_0 \left(1 + \frac{g_0}{2} f_0^{\alpha-1}\right) \tag{19}$$

At this level, we note from Eq. (19) that the springs of modulus E_M and E_α act on the material response.

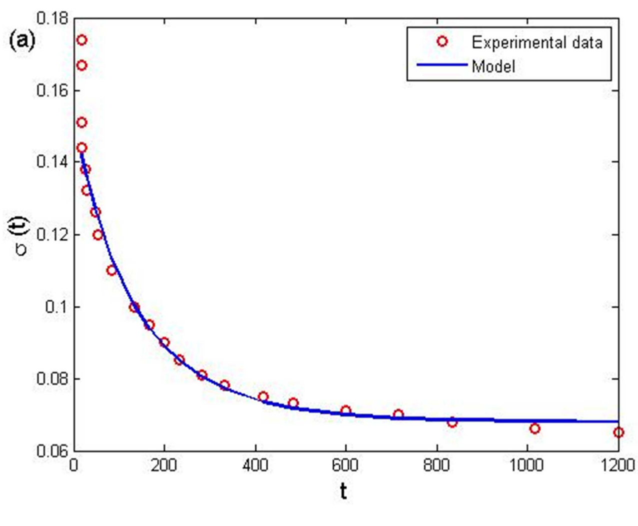
At present, we verify by numerical simulations (Section 3) the capacity of our nonlinear model to reproduce the experimental data of some viscoelastic materials.

3. Validation of the Model

In this section, some numerical examples are given in order to verify the ability of the proposed nonlinear rheological model to describe the stress relaxation data of some viscoelastic materials. To this end, a comparison is performed between the models given by Eqs. (13) and (17) and the stress relaxation data of some viscoelastic materials taken from Liu and Yeung [17], Myhan et al. [18] and Lin [19]. The material parameters E_M, τ_M, g_0, g_1, E_β, β and α used to describe the viscoelastic properties of a variety of viscoelastic materials are determined by fitting of Eqs. (13) and (17) to the stress relaxation data reproduced from Liu and Yeung [17], Myhan et al. [18] and Lin [19]. The curve fitting is performed by using the Trust-Region nonlinear least squares algorithm implemented in MATLAB. The results of the curve fitting are shown in Figures 2-4. The obtained optimal values and adjusted R-square are presented in Tables 1-7. It can be seen from Figures 2-4 that the exact analytical predictions (13) and (17) are an excellent agreement with used experimental data. This agreement is confirmed by the value of adjusted R-square close to one. Therefore, we can conclude that, the proposed rheological model with integer and non-integer orders nonlinearities is capable to reproduce any stress relaxation data of viscoelastic materials.

4. Conclusion

In this study, a single nonlinear differential constitutive equation is derived from a nonlinear rheological model with few constant parameters for predicting the stress relaxation behavior in some viscoelastic materials. Under a constant strain-history, the time-dependent stress is analytically derived in the cases where the positive power law exponent $\alpha < 1$ and $\alpha > 1$. The Trust Region Method implemented in MATLAB Optimization Toolbox is used to determine the optimal values of the material parameters. The exact analytical solutions obtained during the stress relaxation experiment reproduce perfectly the experimental data of some viscoelastic materials available in open literature. The adjusted R-square that justifies the agreement between the exact analytical predictions and experimental data is very close to one. Starting from these obtained results, thedeveloped rheological model with integer and non-integer orders nonlinearities is capable to describe with accuracy the stress relaxation data of viscoelastic solids or fluids. Therefore, we can assert that the pursued research hypothesis in this study is reached.



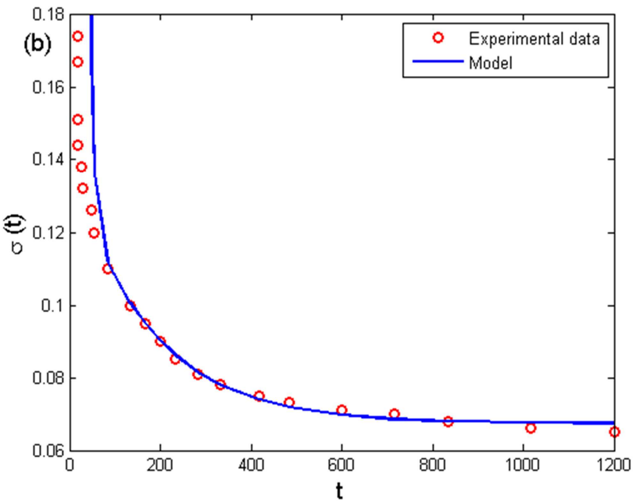


Figure 2. *Time-dependent stress relaxation curves showing the comparison between exact analytical predictions ((a) Eq. (13) and (b) Eq. (17)) and stress relaxation data of the skin tissue taken from Liu and Yeung [17].*

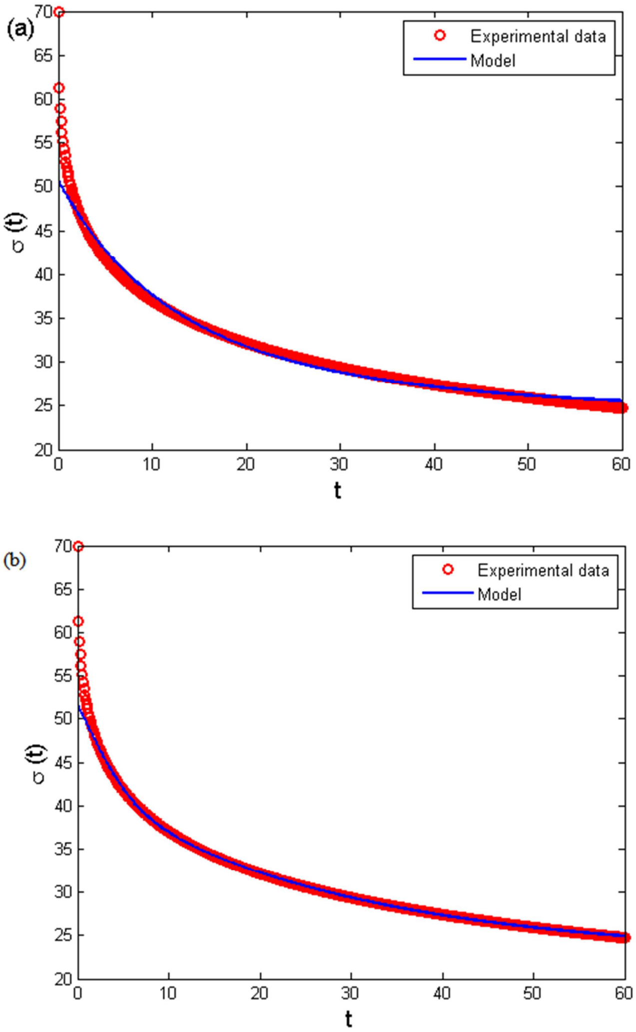


Figure 3. *Time-dependent stress relaxation curves exhibiting the comparison between exact analytical predictions ((a) Eq. (13) and (b) Eq. (17)) and stress relaxation data of canned pork ham taken from Myhan et al. [18].*

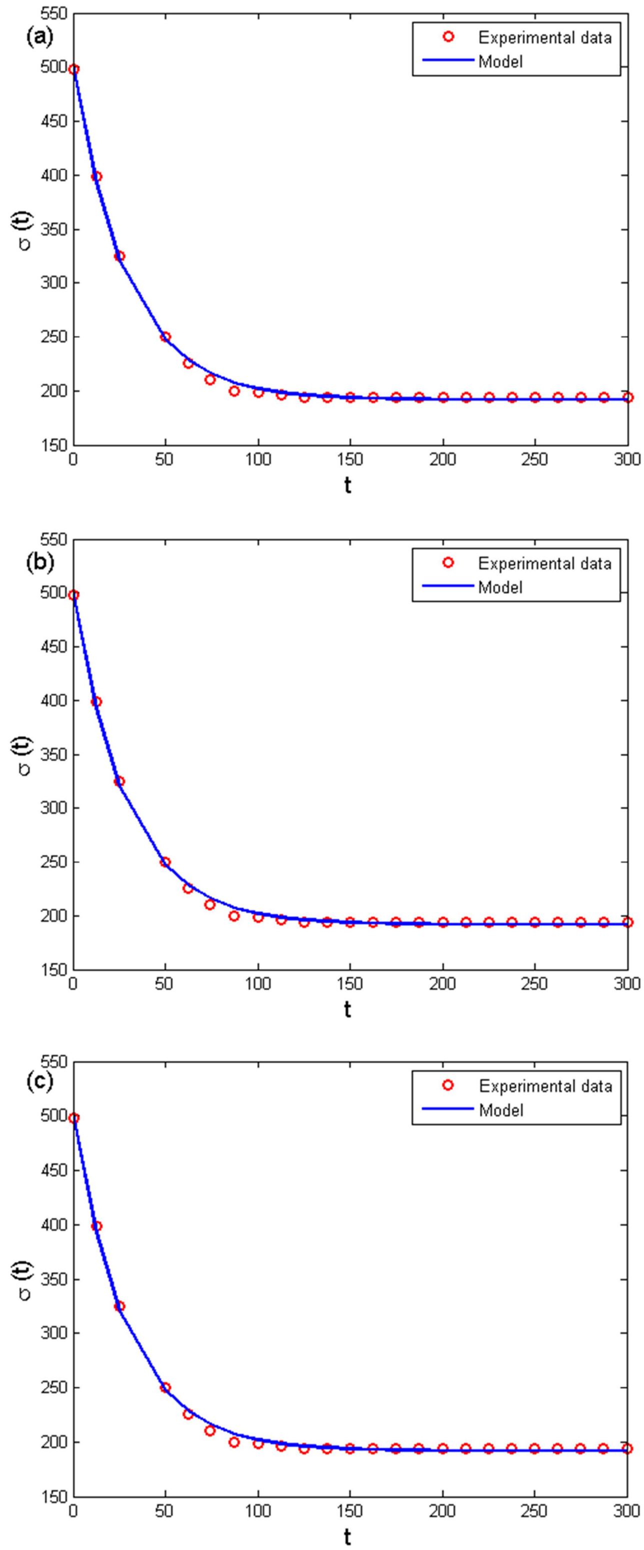


Figure 4. *Time-dependent stress relaxation curves illustrating the comparison between exact analytical predictions ((a) Eq. (13) and (b, c) Eq. (17)) and stress relaxation data of magnetorheological gels taken from Lin [19].*

Table 1. *Parameters obtained by fitting stress relaxation data of the skin tissue (taken from Liu and Yeung [17]) to Eq. (13).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9629	0.2407	34.88	0.08	1.25	0.8	1.321	0.141	0.15

Table 2. *Parameters obtained by fitting stress relaxation data of the skin tissue (taken from Liu and Yeung [17]) to Eq. (17).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9766	0.08467	174	0.9489	1.394	20	140.8	25.41	0.15

Table 3. *Parameters obtained by fitting stress relaxation data of the canned pork ham (taken from Myhan et al.[18]) to Eq. (13).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9917	0.4039	0.8842	34.01	0.4367	0.9702	2.1	0.06537	0.3

Table 4. *Parameters obtained by fitting stress relaxation data of the canned pork ham (taken from Myhan et al.[18]) to Eq. (17).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9993	15.69	27.78	182.9	1.735	11.26	0.2787	0.09342	0.3

Table 5. *Parameters obtained by fitting stress relaxation data of magnetorheological gels (taken from Lin [19]) to Eq. (13).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9984	108.8	26.59	45.06	0.6006	0.1	1.722	2.281	0.05

Table 6. *Parameters obtained by fitting stress relaxation data of magnetorheological gels (taken from Lin [19]) to Eq. (17).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9994	725.4	22.67	194	0.00203	15.32	1.524	1.136	0.05

Table 7. *Parameters obtained by fitting stress relaxation data of magnetorheological gels (taken from Lin [19]) to Eq. (17).*

Material parameters	E_M	$\tau_M(s)$	E_β	$E_\beta(MPa)$	α	g_0	g_1	ε_0
Optimal values Adjusted R-square = 0.9995	332.1	22.35	200	0.0115	2	2.124	0.7383	0.05

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