
Heat Transfer and Solidification Methodology Involved in the Simulation of Steelmaking

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Abstract: The research work done in the last three decades has made continuous casting an advanced and sophisticated technology. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer and structural deformation. The important numerical modeling method of the continuous casting process has been discussed in reference in this work. The present work describes molten steel flow, heat transfer, solidification, formation of the shell by solidification and coupling, etc. Continuous casting process is presently a well-established manufacturing process for steel production. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer, and structural deformation. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. One such kind of method has become more popular to use optimizing using numerical modeling. It describes molten steel flow, formation of the shell by solidification. With the recent advancement in metallurgical methods, the continuous casting process now becomes the main method for steel production. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. In this work, we have studied and reviewed the literature to provide current information on the numerical modeling of continuous casting processes.

Keywords: Steelmaking, Continuous Casting, Numerical Modeling

1. Introduction

With the recent advancement in metallurgical methods, the continuous casting process now becomes the main method for steel production. The purpose for this is the benefits that accompany the nonstop projecting cycle which incorporates cost-saving, high efficiency, and better quality [1–26]. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer and structural deformation. The important part and process of continuous casting have been modeled in-depth and discussed in reference [1]. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. One such kind of method has become more popular to use optimizing using numerical modeling. It describes molten steel flow, formation of the shell by solidification [27]. Further, the distortion of strand by thermo-mechanical forces, bulging, bending and crack prediction has been also given in detail. Till now, many powerful pre-coded solvers are available in the market. The

numerical simulation of the thermo-mechanical behavior of the continuous casting process is important in terms of achieving a quality product [2, 3, 13, 20–26]. The research work done in the last three [11, 18, 27–32] decades has made continuous casting an advanced and sophisticated technology [1, 33–35]. Physical water models can simulate the molten steel flow in the mold region of the continuous casting process considering the viscosity of water equivalent to steel [36–40]. This part of simulation comes with many obstacles such as dealing with the highly non-linear constitutive laws of structure, incorporation of latent heat, involvement of three different states of material: liquid, mushy and solid, temperature-dependent material properties, irregular contact between the mold surface and solidified strand, and coupling the heat transfer and structure model with proper continuum mechanism and boundary condition [41]. Reynold's Averaged Navier–Stokes (RANS) method has been widely adopted for turbulence modeling. It has been reported that the RANS model is highly accurate in predicting steady-state flow patterns [22]. Several research works have been done on

molten steel flow, heat transfer and solidification in mold [38–40, 42–44]. These studies have been established and validated with industrial trials [4, 33, 45–48]. From all previous studies, it is well established that numerical models efficiently and accurately predict the fluid flow and mechanical behavior of mold and strand, respectively [36, 37]. The most adopted technique for simulating the solidification of continuous casting is the enthalpy-porosity approach [10, 12, 14, 15, 20, 22, 28, 49–53]. This technique is based on a quantity known as a liquid fraction. Many researchers have studied this approach but most of them are limited to 2D modeling. Though modes of heat transfer in the mold are complex phenomena and are studied where modes of heat transfer are conduction, convection & radiation, their effect on the final product and the possibility of controlling the detrimental effect has to be studied. In this paper, we have studied and reviewed the literature to provide current information on the numerical modeling of continuous casting processes.

2. Equations for Heat Transfer and Solidification

The fundamental requirement of the continuous casting process is to solidify the strand to achieve plant set quality standards [1, 35, 54–62]. Generalized heat transfer equation (3-dimension) can be written in the most suitable format from the above equations in the following manner [11, 28, 29, 31, 49, 63–77];

$$\rho C \left(\frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z} \right) = \ddot{q} + \frac{\partial}{\partial x} \left(K_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial T}{\partial z} \right) \quad (1)$$

In 2005, Louhenkilpi *et al.* proposed a three-dimensional transient formulation for temperature distribution over the mold wall. [78];

$$\rho \frac{\partial H}{\partial t} + v \frac{\partial H}{\partial z} = \frac{\partial}{\partial x} \left(k_{\text{eff}} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_{\text{eff}} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_{\text{eff}} \frac{\partial T}{\partial z} \right) \quad (2)$$

In a similar work, Zhao *et al.* (2005) [79] modeled energy equation along with the Navier-Stokes equation.

$$\frac{\partial T}{\partial t} + \frac{\partial(u_i T)}{\partial x_i} = \frac{k}{\rho_0 c_p} \frac{\partial}{\partial x_i} \left(\frac{\partial T}{\partial x_i} \right) + \frac{\partial Q_{Ti}}{\partial x_i} \quad (3)$$

$$Q_{Ti} = T u_i - \overline{T u_i} \quad (4)$$

$$\frac{\partial Q_{Ti}}{\partial x_i} = \frac{\mu_T}{Pr_T} \frac{\partial}{\partial x_i} \frac{\partial T}{\partial x_i} \quad (5)$$

In 2011, Sowa and Bokota [80] proposed a heat flow model based on the Fourier-Kirchhoff system of equations.

$$\rho c \left(\frac{\partial T(x,t)}{\partial t} + \nabla T \cdot v \right) = \nabla \cdot (\lambda \nabla T) + \dot{Q} \quad (6)$$

Sowa and Bakota *et al.* [80] modified the above equation

which includes effective specific heat (C_{eff}) term which is a function of the temperature of the material.

$$\nabla \cdot (\lambda \nabla T) - C_{\text{eff}} \frac{\partial T}{\partial t} - C_{\text{eff}} \nabla T \cdot V = 0 \quad (7)$$

$$C_{\text{eff}}(T) = \rho_{LS} c_{LS} + \rho_S L / (T_L - T_S) \quad (8)$$

In 2011, Hadata *et al.*, [43] proposed a steady Fourier-Kirchhoff model for heat flow with some assumptions.

$$q_v = Q_s \frac{dV_s}{d\tau} \quad (9)$$

In a study in 1993 S. E. Chidiac *et al.*, [64] used enthalpy approach for heat transfer in multi-dimensional problem with following equation.

$$\rho \frac{\partial H}{\partial t} = \nabla \cdot (K \nabla T) + Q \quad (10)$$

where ρ indicates density, H indicates enthalpy, K indicates Thermal conductivity, Q indicates heat generation rate for unit volume, T indicates temperature and t time. Enthalpy is nothing but the summation of sensible & latent heat and can be expressed as:

$$H = \int_{T_r}^T c dT + f(T) \cdot L \quad (11)$$

where c , $f(T)$ and L are specific heat liquid fraction and latent heat. For phase change study two methods are clubbed together with the above-stated formulation for accuracy and efficiency. Dirichlet & Cauchy boundary conditions are used to solve above equations. The study carried in 2003, B. wiwanapataphee *et al.*, [63] for simulating phase change cause of heat transfer single domain enthalpy method is adopted. Where enthalpy is the summation of latent heat (H) & sensible heat (h).

$$H = h + \Delta H \quad (12)$$

Latent heat h can be given by

$$H = f(T) L, \quad (13)$$

Where L denoted Latent Heat of Steel L and $f(T)$ indicates localized liquid fraction where value one represents complete Liquids state and zero represents the complete solid-state. The liquid fraction is nonlinear for simplification of the model it is assumed linear.

$$f(T) = \begin{cases} 0, & T \leq T_s \\ \frac{T - T_s}{T_L - T_s}, & T_s < T < T_L \\ 1, & T \geq T_L \end{cases} \quad (14)$$

where in T_L indicates melting temperature and T_s Solidification temperature.

For region where phase change occurs conservation of energy principle. Combining this equation with enthalpy gives,

$$\rho c \left(\frac{\partial T}{\partial t} + u_j T \right) = (k_0 T_j)_j - S_T \quad (15)$$

Ivanova (2013) [81] formulated extensive mathematical modeling on predicting phase-dependent boundary conditions.

$$\frac{\partial T}{\partial r} + v(t) \cdot \frac{\partial T}{\partial z} = \frac{1}{c(T)\rho(T)} \times \left\{ \frac{\partial}{\partial x} \left[\lambda(T) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial z} \left[\lambda(T) \frac{\partial T}{\partial z} \right] \right\} \quad (16)$$

The position of the unknown phase boundary is specified by the equality condition of the temperatures and the Stefan condition for the two-dimensional case:

$$T = T(\tau, x, z)|_{x=\xi(\tau, z)} = T(\tau, x, z)|_{x=\xi_+(\tau, z)=T_{cr}} \quad (17)$$

$$\lambda(T) \frac{\partial T}{\partial n} |_{\xi_+} - \lambda(T) \frac{\partial T}{\partial n} |_{\xi_-} = \mu \rho (T_{kp}) \left(\frac{d\xi}{d\tau} + v(\tau) \frac{d\xi}{dz} \right) \quad (18)$$

where ξ is the phase boundary $x = \xi(\tau, z)$, n is a normal to the phase boundary, $\frac{\partial T}{\partial n} |_{\xi_{+/-}}$ is the left-right limit of the temperature derivative in the normal direction. μ is the latent heat of crystallization. T_{cr} is the crystallization temperature (the average temperature from the liquidus-solidus interval).

In 2014, Zhang et al [82] investigated a steady-state two-dimensional numerical model based on the assumption of heat transfer.

$$\rho = (1 - f_s)\rho_L + f_s(f_\delta\rho_\delta + f_\gamma\rho_\gamma) \quad (19)$$

$$\lambda = (1 - f_s)\lambda_L + f_s(f_\delta\lambda_\delta + f_\gamma\lambda_\gamma) \quad (20)$$

$$c_{eff} = f_s \cdot c_s + (1 - f_s) \cdot c_L - L \frac{\partial f_s}{\partial T} \quad (21)$$

In a similar work, Maurya and Jha (2014) [83] investigated the effect of casting speed in the continuous casting process.

$$\rho \frac{\partial H}{\partial t} + \rho \nabla \cdot (uH) = \nabla(k_{eff} \nabla T) + Q_\ell \quad (22)$$

Where ρ is density, H is enthalpy, ΔH is sensible heat, Q_L is source term. Q_L can be expressed as a single solidification model and given as;

$$Q_L = \rho L \frac{\partial f_s}{\partial t} + \rho L u_{pull} \cdot \nabla f_s \quad (23)$$

$$\frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad (24)$$

Naiver-Stokes equation for transient momentum conservation is given by

$$\frac{\partial}{\partial t} (\rho u) + \rho \nabla(uu) = -\nabla P + \nabla\{\mu_{eff}(\nabla \cdot u)\} + \rho + S_{(xx)} \quad (25)$$

where,

$$\mu_{eff} = \mu_l + \mu_t$$

Maurya and Jha (2014) [83] and Hitanen et al. (2017) [84] used the enthalpy-porosity technique for solidification.

$$S = \frac{(1-\beta)^2}{(\beta^3 - \xi)} A_{mush} (u - u_{pull}) \quad (26)$$

where, liquid fraction is expressed as β , $\xi = 0.001$, mushy zone constant is given as A_{mush} .

Pilvi et al., (2017) [65] Used turbulent flow modelling at inlet in which they considered hydraulic diameter at inlet.

$$\lambda_e = \lambda_L L (1 + 6(1 - f_s)^2) \quad (27)$$

In 2016, Hibbeler et al. [85] proposed an innovative reduced-order model (ROM) for heat transfer from mold in the continuous casting of steel.

$$0 = \frac{\partial^2 \theta_{mould}}{\partial x^{*2}} + \left(\frac{d_{mould}}{w_{mould}} \right)^2 \frac{\partial^2 \theta_{mould}}{\partial y^{*2}} + \left(\frac{d_{mould}}{\ell_{mould}} \right)^2 \frac{\partial^2 \theta_{mould}}{\partial z^{*2}} \quad (28)$$

Vnnysy and Saleem (2017) [86] formulated a mathematical formulation for explicitly calculating the geometrical range of the mushy zone.

$$\rho c_p V_{cast} \frac{\partial T}{\partial z} = \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) - \rho V_{cast} \Delta H_f \frac{\partial \chi}{\partial z} \quad (29)$$

where

$$k = \chi k_1 + (1 - \chi) k_s$$

$$c_p = \chi c_{pl} + (1 - \chi) c_{ps}$$

A decoupled three-dimensional mathematic model of fluid flow and heat transfer in continuous casting billet mould was developed by An et al., (2018) [87].

$$\frac{\partial}{\partial t} (\rho H) + \frac{\partial}{\partial x_j} (\rho \mu_j H) = \frac{\partial}{\partial x_j} \left[\left(\lambda + C_p \frac{\mu_t}{\sigma_t} \right) \frac{\partial H}{\partial x_j} \right] \quad (30)$$

Ole Richter et al. (2017) [88] studied the development of free surface flow for the liquid and/or solid phase change. They considered enthalpy-porosity and volume-of-fluid (VOF) method.

$$\alpha_1 = \begin{cases} 0 & = \text{gas} \\ 0 < \alpha_1 < 1 & = \text{cell contains the interface} \\ 1 & = \text{solid or liquid PCM} \end{cases} \quad (31)$$

The molten steel fraction was completely dependent on the thermal condition (T) of liquid metal. TS and TL indicates same respectively. This can be expressed as follows [88];

$$\gamma_{1,l} = \begin{cases} 0 & \text{if } T < T_s \\ \frac{T - T_s}{T_L - T_s} & \text{if } T_s \leq T \leq T_L \\ 1 & \text{if } T > T_L \end{cases} \quad (32)$$

Where one indicates complete liquid state and zero indicates complete solid state. In between values of solid fraction indicates mushy zone.

In the given formulation the density ρ , the heat capacity c_p , the heat conduction λ and the viscosity μ can be expressed as follows;

$$\rho = \alpha_1 (\gamma_{1,l} \rho_{1,l} + \gamma_{1,s} \rho_{1,s}) + \alpha_2 \rho_2 \quad (33)$$

$$c_p = \alpha_1 (\gamma_{1,l} c_{p1,l} + \gamma_{1,s} c_{p1,s}) + \alpha_2 c_{p2} \quad (34)$$

$$\lambda = \alpha_1(\gamma_{1,l}\lambda_{1,l} + \gamma_{1,s}\lambda_{1,s}) + \alpha_2\lambda_2 \quad (35)$$

$$\mu = \alpha_1\mu_{1,l} + \alpha_2\mu_2 \quad (36)$$

In above equations, the subscripts []_{1,l}, []_{1,s} and []₂ illustrate the property of the bulk liquid, solid and gas phase, respectively. In order to consider natural convection in proposed numerical formulation, the Boussinesq approach was used. Further, the buoyancy modified density ρ_b can be defined as;

$$\rho_b = \alpha_1(\gamma_{1,l}\rho_{1,l}(1 - \beta(T - T_L)) + \gamma_{1,s}\rho_{1,s}) + \alpha_2\rho_2 \quad (37)$$

3. Thermo-mechanical Deformation

The behavior of metal especially steel at high temperature becomes sensitive to strain rate and temperature. Therefore, process design of hot metal working of steel is significantly affected by non-linear behavior of steel. Structural distortion arises in mold and strand due to thermal distribution, which causes thermal stress, cracks and ultimately affects quality strand [40, 89]. Many research has been done on mould thermal distortion in mould and strand [4, 5, 90–92]. In 2006, To measure surface temperature and shell thickness, finite point method was used by Alizadeh *et al.* [3]. It has been also reported heat transfer rate is affected by mold distortion [90, 93]. Many research has been done on mould thermal distortion in mould and strand [90, 91]. Generally, the heat transfer equation is solved with interfacial heat flux data and it is quantified from plant data. Subsequently, equations related to thermo-mechanical distortion in mold and strand is calculated.

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (38)$$

where $d\varepsilon_{ij}^e$ and $d\varepsilon_{ij}^p$ are the incremental elastic and plastic components of the total strain vector $d\varepsilon_{ij}$

In this work they proposed incremental stresses and strains during plastic flow;

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial Y}{\partial \sigma_{ij}} \quad (39)$$

where $d\lambda$ is a scalar multiplying factor, dY is derivative of yield stress and σ_{ij} is the deviatoric stress vector.

In 2000, Lee *et al.* [94] proposed a modified model of thermo-mechanical deformation in strand. They developed a mathematical model for the coupled analysis. The coupled analysis consisted of various mathematical models. The coupled model considered molten steel flow characteristics in mould. Further, it coupled the and heat transfer, thermo-mechanical deformation behavior of a solidifying strand in the continuous casting process. Moreover, Von-mises yield function and associated flow were assumed for increment of stress. The stress in thermo-elasto-plastic material can related as;

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^p - \varepsilon_{kl}^{Th}) \quad (40)$$

where C_{ijkl} , ε_{kl} , ε_{kl}^p , and ε_{kl}^{Th} are the elastic constitutive matrix, total infinitesimal strain, plastic strain, and thermal

strain, respectively.

In a similar work, Ha *et al.*, (2000) [55] carried a mathematical modeling for heat transfer study in secondary cooling zone of continuous casting strand. It was reported that creep was dominant factor in bulging defect. The elastic-plastic creep model for the strand is given by:

$$\dot{\varepsilon} = \alpha \sigma^m \quad (41)$$

where σ (kg/cm²) and $\dot{\varepsilon}$ (1/s) denote the equivalent stress and the creep strain rate, respectively, and m is a constant of 3.15. Also

$$\alpha = 0.0806 \exp \left\{ -\frac{28392}{T+273} \right\} \quad (42)$$

In 2004, Bellet *et al.* [5] introduced a global non-steady state (GNS) method for liquid-solid constitutive model which considered mushy zone during solidification. They reported the following relationship for total strain calculation in liquid and mushy zone;

$$\dot{E} = \dot{\varepsilon}^{vp} + \dot{\varepsilon}^{th} \quad (43)$$

where $\dot{\varepsilon}^{vp}$ is a strain in visco-plastic condition and $\dot{\varepsilon}^{th}$ strain due to thermal expansion. In addition to this, a thermo-elastic-viscoplastic model was used to represent the behavior in the solid state. It was described by the following equations [5];

$$\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{vp} + \dot{\varepsilon}^{th} \quad (44)$$

In a similar work, Liu and Zhu (2006) [95] assumed mould copper plate should exhibit thermoelastic behavior and thermoelastic-plastic behaviour for strand. The isotropic linear elastic stress-strain relation was expressed by the constitutive equation as follows:

$$\sigma_{ij} = 2G\varepsilon_{ij} + [\lambda\varepsilon_{kk} - (3\lambda + 2G)\alpha\Delta T]\delta_{ij} \quad (45)$$

It was reported that the total strain can be expressed as the sum of an elastic strain, a thermal strain, and a plastic strain as follows;

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p + \varepsilon_{ij}^T \quad (46)$$

where, temperature change ΔT may induce a thermal strain of a magnitude.

$$\varepsilon_{ij}^T = \alpha\Delta T\delta_{ij} \quad (47)$$

In a recent work, Li *et al.* (2017) [96] reported that in the mushy zone, the stress in solid steel is supposed to increase linearly with the rise in solid fraction between zero strength temperature (TZST) at $f_s=0.75$ and zero Xdeformation temperature (TZDT) and it can be given as follows;

$$\sigma_s * \frac{(f_s - f_{ZST})}{1 - f_{ZST}} \quad (48)$$

where f_s is solid fraction stress, f_{ZST} is stress at zero strength temperature.

Several authors have predicted the probability of crack formation in solid strand by crack susceptibility coefficient

SC as follows [94, 97, 98];

$$\begin{aligned} S_C &= \frac{Y_M}{Y_C} \text{ for } T f_S \leq f_S < 1 \\ &= 0 \text{ for } 0 \leq f_S < T_S \\ &= 0 \text{ for } Y_M \leq 0 \end{aligned} \quad (49)$$

$$\Delta\{\varepsilon\} = \Delta\{\varepsilon\}_e + \Delta\{\varepsilon\}_p + \Delta\{\varepsilon\}_T(XX) \quad (50)$$

where ε_e elastics strain, ε_p Plastic Strain, ε_T Thermal strain. Thermal strain is given by

$$\Delta\{\varepsilon\}_T = \left(\{\alpha\} + d \left[\frac{D[C]^{-1}\{\sigma\}}{dT} \right) dT \right. \quad (51)$$

where α indicates coefficient of thermal expansion. Further in elastic region stress given by (XX)

$$\Delta\{\sigma\} = [D]_e (\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_T) \quad (52)$$

where D_e Indicates Elastic-Plastic matrix. σ Indicates stress. Further in the plastic region the stress is given by

$$\Delta\{\sigma\} = [D]_{ep} (\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_T) + \Delta\{\sigma\}_T \quad (53)$$

Hence the thermal stress is

$$\Delta\{\sigma\}_T = \frac{[D]_e (\partial\sigma/\partial H) / (\partial\{\sigma\}/\partial T) dT}{H' + \{\sigma\sigma/\partial\{\sigma\}\}^T [D]_{ep} (\partial\sigma/\partial\{\sigma\})} \quad (54)$$

where $\bar{\sigma}$ indicates equivalent stress at node.

They noted that near the meniscus liquid fraction is more compared to bottom slab. It shows that solidification is start early at bottom side. Because of uneven temperature in slab leads to thermal strain which creates thermal stress.

Hadata et al., [11] studied surface crack defect evaluation four criteria used namely plastic work criteria, Rice and Tracy Criteria, modified Rice and Tracy criteria and Latham criteria. Plastic work criteria can be given by following equation

$$C_{EP} = \int_0^1 \varepsilon \sigma dt \text{ for } \sigma_m > 0 \quad (55)$$

where ε indicates strain rate, σ indicates stress. This criteria based on assumption that crack will get generated if strain energy is more than critical value C_{EP} . Plastic strain is evaluated only in region where mean stress is positive.

Following is the criteria given by Rice & Tracy

$$C_{RT} = \varepsilon \exp \left(-\frac{3}{2} \frac{\sigma_m}{\sigma} \right) \quad (56)$$

where σ_m is mean stress & ε indicates strain. This criteria assumes that crack will appear if strain increases beyond C_{RT} .

Following is the criteria given by modified which uses only positive values of strain for calculation of critical parameter C_{RM} . The

$$C_{RM} = \sum \Delta \varepsilon \exp \left(-\frac{3}{2} \frac{\sigma_m}{\sigma} \right) \text{ for } \sigma_m > 0 \quad (57)$$

For Latham Criteria equation is as follows

$$C_{LO} = \int_0^t \sigma_{\max} \dot{\varepsilon} dt \text{ for } \sigma_m > 0 \quad (58)$$

4. Conclusions

Recently, Chen et al. (2019) [99] investigated the mold level fluctuations. These fluctuations are caused by transient bulging of the solidifying shell. Consequently, transient bulging phenomenon affects the quality of the steel. They developed a 1D and 2D model for strand simulation. They reported that mold level fluctuations are highly caused by dynamic bulging. Several constitutive models have been adopted for simulating the solidification stresses using the simple elastic-plastic models [100, 101]. Many literatures have reported about strand bulging between rolls which have caused transverse cracks, radial streaks and centerline macrosegregation [4, 5, 92]. Risso et al. [102] evaluated the thermal stress and strain in the solidifying shell of the strand by using the analytical method. Researchers added a separate creep model for transient modeling [103]. The integration of these transient constitutive laws and further, mathematical modeling is a challenging task. From all the above discussion it is observed that the temperature and stress-strain distribution in the strand region of the continuous casting process plays an important role in defining the quality of the final solidified product [41, 104–106]. A numerical model was presented by Fachinotti et al. (2006) [92] to study the macro-segregation defects in strand caused by thermal stress. They made a hypothesis about the transient effect of alternate rolling and bulging. To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [3]. They compared FPM results with FVM results. It was concluded that heat transfer, surface temperature, and shell thickness can be successfully modeled by FPM method. In 2006, Liu and Zhu [95] developed a three-dimensional finite-element heat-transfer and thermal stress models to study the thermo-mechanical distortion on the slab during operation. They reported that operating parameters i.e., casting affected the strand distortion in copper walls of the mould. Pascon and coworkers (2006) [107] studied the generation of transverse crack during bending and straightening of strands. The numerical model was applied and validated with industrial data. The transverse cracks were found at the upper face of the strand.

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